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Cosmology of intersecting brane world models in Gauss-Bonnet gravity

Hyun Min Lee* **and Gianmassimo Tasinato†**

Physikalisches Institut der Universität Bonn
Nussallee 12, D-53115 Bonn, Germany

ABSTRACT: We study the cosmological properties of a codimension two brane world that sits at the intersection between two four branes, in the framework of six dimensional Einstein-Gauss-Bonnet gravity. Due to contributions of the Gauss-Bonnet terms, the junction conditions require the presence of localized energy density on the codimension two defect. The induced metric on this surface assumes a FRW form, with a scale factor associated to the position of the brane in the background; we can embed on the codimension two defect the preferred form of energy density. We present the cosmological evolution equations for the three brane, showing that, for the case of pure AdS_6 backgrounds, they acquire the same form of the ones for the Randall-Sundrum II model. When the background is different from pure AdS_6, the cosmological behavior is potentially modified in respect to the typical one of codimension one brane worlds. We discuss, in a particular model embedded in an AdS_6 black hole, the conditions one should satisfy in order to obtain standard cosmology at late epochs.

*minlee@th.physik.uni-bonn.de
†tasinato@th.physik.uni-bonn.de

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1. Introduction

Status of cosmology on codimension two brane worlds

In the past few years, the possibility that the observed world is localized on a brane, embedded in a higher dimensional space, has received much attention, opening new possibilities for phenomenology [1, 2], and furnishing interesting connections with string theory.

In particular, cosmological aspects of brane-world models in codimension one defects have been extensively analyzed, due to the fact that, in this case, it is possible to compute the backreaction of the brane in the bulk geometry, using the so-called Israel-Lanczos junction conditions. The most studied models for cosmology, based on the Randall-Sundrum II background, are characterized by standard cosmological evolution at late times, while corrections to the standard Hubble equation, due to modification of gravity, occur at early cosmological epochs.

The interesting properties of brane world models on codimension two defects (for example, three branes on six dimensional space-times) have been recently re-considered, for the

possibility to construct new examples of warped brane world scenarios in six dimensions [3], or to face the cosmological constant problem from a new perspective [4] (see however [5]). Less attention has been dedicated to study cosmological aspects of brane worlds for codimension higher than one, since it is in general difficult to take into account in a consistent way the backreaction of the brane in this case. Relatively to this problem, it has been shown long ago [6] how to embed consistently a brane, characterized by pure tension, on the tip of a conical singularity of the higher dimensional geometry (see also [7] for related examples). The subsequent problem to embed more general energy density on this brane has been considered [8], with the quite surprising result that, with the most natural ansatz for the six dimensional metric, it is *not* possible to localize energy density that is different from pure tension. A way out for this unpleasant situation has been presented in [9], where the authors observe that a Gauss-Bonnet term, added to the initial six dimensional action, induces new terms on the brane junction conditions that allow to embed the preferred matter on the conical defect.

A new approach

In this paper, motivated by the ideas presented in [9] and [10], we would like to present an alternative approach to construct cosmological models on codimension two defects, in the framework of theories with Gauss-Bonnet terms. Our approach is different from brane worlds on the tip of a conical singularity: in our scenario, a codimension two defect (a three brane) is located at the *intersection* between two codimension one, four branes at right angle, in a six dimensional space. Each of these four branes corresponds to a fixed hypersurface for a Z_2 symmetry, that identifies the space on the two sides of the hypersurface. The resulting geometry does not present conical singularities. We show that, in the framework of Gauss-Bonnet gravity, it is possible to consistently work out the junction conditions at the intersection, to localize the preferred energy density on this codimension two defect, and, in particular cases, to obtain a (late time) standard cosmology, similarly to well-known codimension one examples in five dimensions.

The main motivation is to construct examples of cosmological models for our codimension two brane worlds, in which also the background geometry (bulk plus codimension one four branes that intersect) is completely specified. We have a complete control of the evolution equations for the codimension two brane world, and we relate this evolution to a time dependence of the system of codimension one branes. This in general requires a degree of fine tuning between energy density on the codimension two brane and the one on the codimension one branes. It would be interesting to understand whether also other approaches for constructing cosmological models in codimension two defects require similar fine tunings.

An important property of our framework consists on the fact that it is not necessary to consider solutions for Einstein-Gauss-Bonnet theory that present conical singularities. Indeed,

any bulk solution is suitable to embed codimension one branes that intersect, providing the codimension two brane-world we are interested in. Moreover, our approach could suggest a way to obtain models for cosmology based on the picture of intersecting brane worlds, where the full backreaction of the branes is taken into account. This kind of configurations receive large attention in String Theory, for the possibility to obtain models with low energy Standard Model-like spectra [11].

The models

We discuss brane world models defined in vacua for six dimensional Einstein-Gauss-Bonnet gravity with negative cosmological constant [1]. We embed in these space-times two codimension one, four branes that intersect at right angle, working out the Israel-Lanczos junction conditions. We show that, in order to satisfy these conditions, energy density must be localized on a codimension two defect located at the intersection between the four branes. We focus on the case in which one of the four branes *moves* through the static AdS_6 background, in the mirage cosmology framework of [15], applied to Z_2 symmetric brane worlds in [16]. The intersection between the branes moves as well: we define our 3+1 dimensional brane world on this surface, and consequently we provide an example of codimension two brane world that moves through the six dimensional background. One finds that the energy density localized on the codimension two brane acquires different forms depending on how the energy momentum tensor of the moving codimension one four brane is chosen. Looking on the other way around, we can choose our preferred form for the energy density localized on the intersection, and consequently define the energy momentum tensor on the four brane that moves.

In particular, we consider two models. The first one is a brane world system embedded in a pure AdS_6 background: working out in detail the evolution equations for the projected cosmology in the codimension two defect, we show that the projected Hubble equation acquires *exactly* the same structure of the Hubble equation in the five dimensional Randall-Sundrum II model. At late epochs the projected Friedmann equation in the codimension two brane approaches the one of standard cosmology: this indicates that, at large distances, gravity behaves as four dimensional, without the necessity of compactifying the extra dimensions. We point out that, as expected, a certain degree of fine tuning is required between the energy density localized on the three brane at the intersection, and the energy density on the moving four brane. In the second model, we use as background geometry an AdS black hole in six dimensions. We embed also in this case two four branes that intersect at right angle, and we limit our attention to the cosmology on the codimension two intersection. The analysis of this

[1] Brane-world models embedded in five dimensional vacua for Einstein-Gauss-Bonnet gravity have been extensively studied [12, 13]; less effort has been devoted to construct models in six dimensions, with the exception of [10, 14].

more general system renders manifest interesting differences between the evolution equations of our codimension two brane world, and the traditional codimension one brane worlds. We find that, in this case, the resulting cosmological properties are in general non standard: the equation of continuity for the energy density is not satisfied, and the effective Planck mass varies with time. These non standard features are related to the geometrical properties of the background. We discuss the conditions to recover, at least at late epochs, a standard cosmological behavior.

2. Formalism: Branes in six dimensional Gauss-Bonnet gravity

Let us start presenting the general action for our six dimensional system

$$S = M_6^4 \int d^6x\sqrt{-g}\left(\frac{1}{2}R - \mathcal{L}_6 + \frac{1}{2}\alpha\mathcal{L}_{GB}\right), \tag{2.1}$$

where M_6, $\mathcal{L}_6$ and α are the six dimensional fundamental mass scale, the Lagrangian for bulk fields, and the Gauss-Bonnet coupling with mass dimension -2, respectively. The Gauss-Bonnet term is defined in six dimensions as [2]

$$\mathcal{L}_{GB} = R^2 - 4R_{MN}R^{MN} + R_{MNPQ}R^{MNPQ}. \tag{2.2}$$

The Einstein equations relative to this action are given by

$$G_{MN} + 2\alpha H_{MN} = T_{MN} \tag{2.3}$$

with T_{MN} the energy momentum tensor relative to the bulk Lagrangian $\mathcal{L}_6$, and

$$\begin{aligned} H_{MN} =& RR_{MN} - 2R_{MP}R^P{}_N - 2R^{PQ}R_{MPNQ} + R_M{}^{PQR}R_{NPQR} \\ &- \frac{1}{4}g_{MN}\mathcal{L}_{GB}. \end{aligned} \tag{2.4}$$

Considering a vacuum for this theory, we can embed two codimension one branes (four branes) Σ_1 and Σ_2 intersecting at right angle, corresponding to fixed points of two Z_2 symmetries that identify the space in each side of them. It is possible to localize fields in their world volume, that are described by the action

$$S_{br} = \int_{\Sigma_1} d^5x\sqrt{-h_1}\mathcal{L}_1 + \int_{\Sigma_2} d^5x\sqrt{-h_2}\mathcal{L}_2 + \int_{\Sigma_3\equiv\Sigma_1\cap\Sigma_2} d^4x\sqrt{-h_3}\mathcal{L}_3. \tag{2.5}$$

[2]This term, that in six dimensions does not correspond to a topological invariant, has been actually first discussed by Lovelock [17]. We thank V. Rychkov for this terminological remark.

where h_i denote the projected metrics on the subspaces, and we include possible contributions localized on a codimensions two defect at the intersection between the branes.

The junction conditions for each of the two codimension one branes are dictated by the Israel junction conditions, extended to the Gauss-Bonnet case in [18]. Let us present the junction conditions on the brane Σ_2: the ones for the brane Σ_1 are analogous, and both sets of conditions must be satisfied simultaneously. They are given by the equations

$$2\langle K_{ab} - K h_{ab}\rangle + 2\alpha\langle 3J_{ab} - J h_{ab} + 2P_{acdb}(h)K^{cd}\rangle = -\frac{1}{M_6^4}(\tilde{S}_{ab} + S_{ab}) \tag{2.6}$$

where $\langle X\rangle \equiv (X(\Sigma_{2,+}) + X(\Sigma_{2,-}))/2$, while the induced four brane metric is $h_{MN} = g_{MN} - n_M n_N$, with a unit normal vector n_M to this codimension one defect [3]. The extrinsic curvature tensor evaluated on Σ_2 is given by $K_{ab} = h_a^M h_b^N \nabla_M n_N$ with $K = K^a{}_a$, and the tensors J_{ab} and P_{abcd} are given by

$$J_{ab} = \frac{1}{3}(2KK_{ac}K^c{}_b + K_{cd}K^{cd}K_{ab} - 2K_{ac}K^{cd}K_{db} - K^2 K_{ab}), \tag{2.7}$$

$$P_{abcd} = R_{abcd} + 2R_{b[c}g_{d]a} - 2R_{a[c}g_{d]b} + Rg_{a[c}g_{d]b}\ . \tag{2.8}$$

The energy momentum tensors relative to matter localized on Σ_2 , appearing on the right hand side of (2.6), are given by

$$\tilde{S}_{ab} = -\frac{2}{\sqrt{-h_2}}\frac{\delta\left(\sqrt{-h_2}\mathcal{L}_2\right)}{\delta h_2^{ab}}, \tag{2.9}$$

$$S_{ab} = -\delta(\Sigma_1)\delta_a^\mu\delta_b^\nu\frac{2}{\sqrt{-h_2}}\frac{\delta\left(\sqrt{-h_3}\mathcal{L}_3\right)}{\delta h_3^{\mu\nu}} \equiv \delta(\Sigma_1)\sqrt{\frac{-h_3}{-h_2}}\delta_a^\mu\delta_b^\nu S_{\mu\nu}\ . \tag{2.10}$$

The last quantity S_{ab} denotes energy momentum tensor that is localized on the intersection Σ_3 between the branes. When considering the usual Einstein gravity, one finds that, in this situation, there are no terms, in the geometrical part of junction conditions, that are localized at the intersection. This means that energy momentum tensor S_{ab} should vanish.

This is no more true when we add the Gauss-Bonnet part: contributions contained in the P-tensor, in the left hand side of (2.6), result to be localized just at the intersection between the codimension one branes, and require energy momentum tensor at the intersection in order to satisfy the junction conditions. We will show this fact by explicit examples in the next sections.

[3] Capital letters denote 6d indices, $M, N = 0, 1, 2, 3, 4, 5$. Small letters denote 5d indices, $a, b = 0, 1, 2, 3, 4$. Greek letters denote 4d indices, $\mu, \nu = 0, 1, 2, 3$.

3. First model: Intersecting branes in AdS$_6$

In this section, we deal with the simplest example of bulk action in which the formalism presented in the previous section can be applied: an AdS bulk, where $\mathcal{L}_6 \equiv \Lambda_6$ corresponds to negative cosmological constant, and action given by

$$S = M_6^4 \int d^4x dz_1 dz_2 \sqrt{-g} \left(\frac{1}{2} R - \Lambda_6 + \frac{1}{2} \alpha \mathcal{L}_{GB} \right) \tag{3.1}$$

Then, we can solve the Einstein-Gauss-Bonnet equations (2.3), with $T_{MN} = -\Lambda_6 g_{MN}$, obtaining a warped metric with flat four dimensional subspace

$$ds^2 = A^2(z_1, z_2)(\eta_{\mu\nu} dx^\mu dx^\nu + dz_1^2 + dz_2^2)\,, \tag{3.2}$$

with a warp factor given by [10]

$$A(z_1, z_2) = 1/(k_1 z_1 + k_2 z_2 + 1)\,, \tag{3.3}$$

while the constants k_1 and k_2 satisfy the equation

$$k_1^2 + k_2^2 = \frac{1}{12\alpha} \left[1 \pm \sqrt{1 + \frac{12\alpha\Lambda_6}{5}} \right]. \tag{3.4}$$

The two signs correspond to two branches of solutions: from now on, we consider the minus sign, since in this case taking $\alpha \to 0$ we obtain the correct limit to the AdS solution of standard Einstein gravity [19].

Let us embed in this background two codimension one branes, that intersect with a 90 degree angle. In the next subsection, we review the conditions to have a static system, with the two static branes intersecting at the origin. In subsection (3.2), instead, we present a model in which one of the four branes moves through the bulk, inducing cosmology both on its world-volume, and on the codimension two defect that lives at the intersection with the other four brane.

3.1 Static case

Starting from metric (3.2), let us consider the situation in which the two four branes are situated respectively on the hypersurfaces $z_1 = 0$ and $z_2 = 0$ [4]. Since they constitute fixed points of a Z_2 symmetry, the bulk metric in this case becomes

[4]This static system of branes at right angle has been considered in [10] (see also [20]).

$$ds^2 = A^2(|z_1|,|z_2|)(\eta_{\mu\nu}dx^\mu dx^\nu + dz_1^2 + dz_2^2) \tag{3.5}$$

and the warp factor becomes $A(|z_1|,|z_2|) = 1/(k_1|z_1| + k_2|z_2| + 1)$. The junction conditions at the brane positions force to choose the brane energy densities as pure tensions, that is:

$$\mathcal{L}_1 = -\Lambda_1\,, \quad \mathcal{L}_2 = -\Lambda_2\,, \quad \mathcal{L}_3 = -\lambda\,. \tag{3.6}$$

The tensions must satisfy the relations

$$k_1\left[1 - 12\alpha\left(\frac{1}{3}k_1^2 + k_2^2\right)\right] = \frac{\Lambda_1}{8M_6^4}\,, \tag{3.7}$$

$$k_2\left[1 - 12\alpha\left(k_1^2 + \frac{1}{3}k_2^2\right)\right] = \frac{\Lambda_2}{8M_6^4}\,, \tag{3.8}$$

$$\alpha k_1 k_2 = \frac{\lambda}{96M_6^4}\,. \tag{3.9}$$

The interesting point is that, in this system, the codimension two brane Σ_3 located at the intersection between the four branes is characterized by its own tension λ, proportional to the Gauss-Bonnet parameter α. This means that the presence, in the six dimensional action, of the Gauss-Bonnet term implies the presence of energy density localized on the three brane. We will develop this observation in the context of moving branes in the next subsection.

3.2 Moving branes: Cosmology

Starting from [15], it has been observed that branes that move through a static bulk induce cosmology on their own world volume. The time evolving projected scale factor depends on the position of the brane in the bulk, and the time dependent energy density localized on the brane must satisfy suitable junction conditions, that change with changing the brane position.

Let us apply this idea to our system, demanding that one of the two branes, let us say Σ_2, moves through the static background constituted by the AdS bulk, while the other brane Σ_1 is fixed at the position $z_1 = 0$. In this situation, the three brane that sits on the intersection moves as well, and, as we will see, this generates an induced cosmology also at the intersection.

We start evaluating the junction conditions at the fixed points of the Z_2 symmetries, next we continue discussing the features of the projected cosmology on the three and four branes.

3.2.1 Evaluation of junction conditions

We demand that the four brane Σ_2 moves through the static AdS bulk in which the static Σ_1 brane is located. The starting bulk metric is consequently the following,

$$ds^2 = A^2(|z_1|, z_2)(\eta_{\mu\nu}dx^\mu dx^\nu + dz_1^2 + dz_2^2)\,, \tag{3.10}$$

and the static brane Σ_1 located in $z_1 = 0$, the fixed point of a Z_2 symmetry, satisfies the junction condition (3.7). Calling $z_2(t)$ the position of the moving brane, the induced metric on its world volume results to be

$$\begin{aligned} ds^2_{\Sigma_2} &= A^2(|z_1|, z_2(t))\Big[-(1-\dot{z}_2^2)dt^2 + \delta_{ij}dx^i dx^j + dz_1^2\Big] \\ &\equiv -n^2(|z_1|, t)dt^2 + a^2(|z_1|, t)(\delta_{ij}dx^i dx^j + dz_1^2) \end{aligned} \tag{3.11}$$

where the dot denotes the derivative with respect to t. Then, the bulk space for $z_2 < z_2(t)$ is identified with the one for $z_2 > z_2(t)$ by a Z_2 symmetry.

Let us continue evaluating the geometrical quantities that appear in the left hand side of eq. (2.6), that represent the junction conditions on the brane Σ_2. The results of our calculations show that there are geometrical terms, on this equation, that are localized at the intersection between the four branes, and must be compensated by some form of energy momentum tensor localized on this codimension two surface.

The velocity vector (u^M) and the normal vector (n_M) for the moving 4-brane are given by

$$u^M = \frac{1}{A\sqrt{1-\dot{z}_2^2}}(1, \dot{z}_2, \vec{0})\,, \tag{3.12}$$

$$n_M = \frac{A}{\sqrt{1-\dot{z}_2^2}}(-\dot{z}_2, 1, \vec{0})\,. \tag{3.13}$$

Then, the spatial components of the extrinsic curvature at the 4-brane are given by

$$\begin{aligned} K^{z_1}_{z_1} = K^i_i &= \frac{1}{2}n^{z_2}\partial_{z_2}h_{ij} \\ &= -\frac{k_2}{\sqrt{1-\dot{z}_2^2}} \equiv \mathcal{K}\,. \end{aligned} \tag{3.14}$$

On the other hand, the (00) component of the extrinsic curvature at the 4-brane is given by

$$K^0_0 = -K_{MN}\,u^M u^N = \frac{1}{A^2\dot{z}_2}\frac{d}{dt}\left(\frac{A}{\sqrt{1-\dot{z}_2^2}}\right)\,. \tag{3.15}$$

From the induced metric of the moving 4-brane Σ_2, eq. (3.11), we obtain the non zero components of the Riemann tensor and of the Ricci tensor, respectively, as the following,

$$\begin{aligned} R^{0i}{}_{0j} &= (X - Y - \bar{Y})\delta^i_j, & R^{0z_1}{}_{0z_1} &= X - Y - \bar{X} + \bar{Y}, \\ R^{ik}{}_{jl} &= (Y - \bar{Y})(\delta^i_j\delta^k_l - \delta^i_l\delta^k_j), & R^{iz_1}{}_{jz_1} &= (Y - \bar{X} + \bar{Y})\delta^i_j, \end{aligned} \tag{3.16}$$

$$
\begin{aligned}
R^0{}_0 &= 4(X-Y) - \bar{X} - 2\bar{Y}\,, \\
R^i{}_j &= (X + 2Y - \bar{X} - 2\bar{Y})\delta^i_j\,, \\
R^{z_1}{}_{z_1} &= X + 2Y - 4\bar{X} + 4\bar{Y}
\end{aligned} \tag{3.17}
$$

where we define

$$
X = \frac{1}{n^2}\left[\frac{\ddot{a}}{a} - \frac{\dot{a}}{a}\left(\frac{\dot{n}}{n} - \frac{\dot{a}}{a}\right)\right]\,, \qquad Y = \frac{1}{n^2}\left(\frac{\dot{a}}{a}\right)^2\,, \tag{3.18}
$$

$$
\bar{X} = \frac{1}{a^2}\left(\frac{a''}{a}\right)\,, \qquad \bar{Y} = \frac{1}{a^2}\left(\frac{a'}{a}\right)^2. \tag{3.19}
$$

Then, the divergence-free part Riemann tensor, given in eq. (2.8), is given by

$$
\begin{aligned}
P^{0i}{}_{0j} &= (3Y - 2\bar{X} + \bar{Y})\delta^i_j\,, & P^{0z_1}{}_{0z_1} &= 3(Y - \bar{Y})\,, \\
P^{ik}{}_{jl} &= (2X - Y - 2\bar{X} + \bar{Y})(\delta^i_j\delta^k_l - \delta^i_l\delta^k_j)\,, & P^{iz_1}{}_{jz_1} &= (2X - Y - 3\bar{Y})\delta^i_j\,.
\end{aligned} \tag{3.20}
$$

With these quantities, we can evaluate the geometrical quantities of the left hand side of the junction conditions (2.6). We must now specify the form of the energy momentum tensors that appear in the right hand side of this equation. We impose that the energy momentum tensors, both on the four brane Σ_2 and on the three brane Σ_3, acquire a perfect fluid form:

$$
\tilde{S}^a{}_b = \mathrm{diag}(-\tilde{\rho}, \tilde{p}, \tilde{p}, \tilde{p}, \tilde{p})\,, \tag{3.21}
$$

$$
S^\mu{}_\nu = \mathrm{diag}(-\rho, p, p, p)\,. \tag{3.22}
$$

Then, we can substitute all the quantities on the junction conditions (2.6). The energy density and pressure on the *four* brane Σ_2, $\tilde{\rho}$ and $\tilde{p}$, are given by

$$
\tilde{\rho} = -8M_6^4\mathcal{K}[1 - 4\alpha(3k_1^2 + 3k_2^2 - 2\mathcal{K}^2)]\,, \tag{3.23}
$$

$$
\tilde{p} = 2M_6^4(K_0^0 + 3\mathcal{K})[1 - 12\alpha(k_1^2 + k_2^2)] + 64\alpha M_6^4\mathcal{K}^3 - 16\alpha M_6^4\frac{z_2}{\dot{z}_2}\frac{d}{dt}\mathcal{K}^3\,, \tag{3.24}
$$

while the energy density and pressure on the *three* brane Σ_3, ρ and p, result to be

$$
\rho = -96\alpha k_1 M_6^4\mathcal{K}|_{z_1=0}\,, \tag{3.25}
$$

$$
p = -\rho + 32\alpha k_1 M_6^4(K_0^0 - \mathcal{K})|_{z_1=0}. \tag{3.26}
$$

3.2.2 Cosmology at the intersection

Since we obtained the connection between energy momentum tensor and geometrical quantities, we are ready to discuss the cosmological properties of our background. We first consider the induced cosmology on the codimension two, three brane Σ_3.

Starting from eq. (3.11), a redefinition of the time variable allows to write the induced metric on the 3-brane in a FRW form:

$$ds^2_{3-brane} = -d\tau^2 + R^2(\tau)\delta_{ij}dx^i dx^j \,, \tag{3.27}$$

with

$$R(\tau) \equiv A(z_1 = 0, z_2(\tau)) = \frac{1}{k_2 z_2(\tau) + 1} \,, \tag{3.28}$$

where the proper time for the 3-brane is obtained from the t coordinate as a coordinate transformation,

$$d\tau = A(z_1 = 0, z_2(t))\sqrt{1 - \dot{z}_2^2}\, dt \quad \to \quad \sqrt{1 - \dot{z}_2^2} = \frac{1}{\sqrt{1 + A^2(\frac{dz_2}{d\tau})^2}} \,.$$

The scale factor is related to the position of the codimension two brane in the bulk. Equations (3.25) and (3.26) ensure that the standard continuity equation is satisfied for energy momentum tensor on the three brane, that is,

$$\frac{d\rho}{d\tau} + 3H(\rho + p) = 0 \,. \tag{3.29}$$

Equation (3.25) furnishes us the Friedmann equation on the brane, that acquires the remarkably simple form

$$H^2 = k_2^2 \left[\left(\frac{\rho}{\lambda} \right)^2 - 1 \right] . \tag{3.30}$$

where the tension λ is defined in the eq. (3.9). It is easy to show that (3.30) can be re-casted on the same form of Friedman equation that one finds in the Randall-Sundrum model. Indeed, decomposing

$$\rho \equiv \lambda + \rho_3 \,, \qquad p \equiv -\lambda + p_3 \,, \tag{3.31}$$

and defining

$$M_4^2 = \frac{\lambda}{6k_2^2} = \alpha \frac{16 k_1 M_6^4}{k_2} \,, \tag{3.32}$$

one can rewrite eq. (3.30) as

$$H^2 = \frac{1}{3M_4^2}\rho_3 + \frac{1}{6M_4^2}\frac{\rho_3^2}{\lambda} \,. \tag{3.33}$$

Therefore, we see that in this model we obtain standard cosmology at late times, when $\lambda \gg \rho_3$, while we have corrections to the standard form of the Hubble parameter at early times. The fact that at late epochs the second term in the right hand side of (3.33) becomes negligible, and we obtain the standard form for the Friedmann equation, indicates that gravity on the brane behaves as four dimensional at large distances. Interestingly, notice that in this model the four dimensional, induced Planck mass, given by (3.32), results proportional to the Gauss-Bonnet parameter α, like in the approach of [9].

As anticipated, the form of the Friedmann equation (3.30) is the same as the codimension one, Randall-Sundrum II brane world. This observation will be true only when the background is purely AdS: for a more general bulk, as we will discuss in Section (4), the evolution equations are different in respect to the ones that typically arise in codimension one brane worlds.

3.2.3 Cosmology on the four brane

Following the same procedure of the previous case, we can write the cosmological evolution equation for the moving four brane Σ_2, with induced metric given in eq. (3.11). Equations (3.23) and (3.24) ensure that a continuity equation for energy momentum tensor on the four brane (given in eq. (3.21)) is satisfied:

$$\dot{\tilde{\rho}} + 4\frac{\dot{a}}{a}(\tilde{\rho} + \tilde{p}) = 0\,, \tag{3.34}$$

where we use here the six dimensional time t.

The form of the Friedmann equation on the brane is more complex than the three brane case, and contains various contributions coming from the Gauss-Bonnet terms. It is convenient to define a (position dependent) proper time $\tilde{\tau}$ via the formula

$$d\tilde{\tau} = A(z_1, z_2(t))\sqrt{1 - \dot{z}_2^2}\,dt\,, \tag{3.35}$$

where notice that for the limiting case $z_1 = 0$ this expression coincides with the one we used for the three brane proper time. Defining the four brane scale factor as

$$R(\tilde{\tau}) = A(z_1, z_2(\tilde{\tau}))\,,$$

we obtain the following form for the four brane Friedmann equation:

$$H^2 = \left(\frac{1}{R}\frac{dR}{d\tilde{\tau}}\right)^2 = c_+ + c_- - \frac{1}{12\alpha}\sqrt{1 + \frac{12\alpha\Lambda_6}{5}} - k_2^2\,. \tag{3.36}$$

The quantities c_+ and c_- are given by the following expressions

$$c_\pm = \frac{1}{24\alpha}\left[(1 + \frac{12\alpha\Lambda_6}{5})^{\frac{3}{2}} + \frac{27\alpha}{16M_6^8}\tilde{\rho}^2 \pm \frac{9}{2}\sqrt{\frac{\alpha}{6}}\frac{\tilde{\rho}}{M_6^4}\sqrt{(1 + \frac{12\alpha\Lambda_6}{5})^{\frac{3}{2}} + \frac{27\alpha}{32M_6^8}\tilde{\rho}^2}\,\right]^{\frac{1}{3}}\,. \tag{3.37}$$

It is not easy to extract interesting physical information from this Friedmann equation. In any case, decomposing the energy density and the pressure as

$$\tilde{\rho} \equiv \Lambda_2 + \rho_4\,, \qquad \tilde{p} \equiv -\Lambda_2 + p_4\,, \tag{3.38}$$

and inserting these expressions in the right hand side of eq. (3.36), one finds that, for $\Lambda_2 \gg \rho_4$, the dominant contribution is a *linear* term in the energy density ρ_4.

3.2.4 Fine tuning issues

The motion of the four brane through the bulk is necessarily related to the one of the three brane, since the latter sits on the former. This means that, in the limit $z_1 \to 0$, we should impose that the evolution for the four brane scale factor, ruled by the corresponding Friedmann equation, results the same as the one for the three brane scale factor (that is, the intersection must "follow" the moving four brane).

Calling $(\tilde{\rho}_0, \tilde{p}_0)$ energy density and pressure for the four brane at $z_1 = 0$, straightforward calculations using the junction conditions show that we must impose the following relations between energy densities and pressures between the three and the four brane:

$$\tilde{\rho}_0 = \frac{\rho}{\lambda}\left[\Lambda_2 - 64\alpha k_2^3 M_6^4\left(1 - \frac{\rho^2}{\lambda^2}\right)\right], \tag{3.39}$$

$$\tilde{p}_0 = \frac{1}{4}\tilde{\rho}_0\left(-1 + \frac{3p}{\rho}\right) + 96\alpha k_2^3 M_6^4\left(\frac{\rho}{\lambda}\right)^3\left(1 + \frac{p}{\rho}\right)\,. \tag{3.40}$$

These equations represent fine tuning relations that must be satisfied, in our system, in order to obtain consistent cosmology. It is interesting to see that, at late epochs of cosmological evolution, they acquire a simple, linear form. Indeed, decomposing the energy densities and pressures as in eqs. (3.31) and (3.38), the fine-tuning equations (3.39)-(3.40) become, at linear order,

$$\rho_4 \simeq (\Lambda_2 + 128\alpha k_2^3 M_6^4)\frac{\rho_3}{\lambda}, \tag{3.41}$$

$$p_4 \simeq -\rho_4 + \frac{3}{4}\left(\Lambda_2 + 128\alpha k_2^3 M_6^4\right)\frac{1}{\lambda}(\rho_3 + p_3)\,. \tag{3.42}$$

The ratios between pressure and energy densities, that define the equations of state, are related, in this late epoch limit, via the following expression

$$\frac{p_4}{\rho_4} \simeq \frac{3}{4}\left(\frac{p_3}{\rho_3} - \frac{1}{3}\right)\,. \tag{3.43}$$

This is the expression one expects to obtain when the Friedmann equations for the three and the four brane are dominated by the term linear in the respective energy density.

4. Second model: Intersecting Branes in AdS Black Hole

In Einstein-Gauss-Bonnet theories with negative cosmological constant, the most general solution for the equations of motions relative to the system of eq. (3.1), with a spherical metric ansatz, is given in [21]. This is given by

$$ds_6^2 = -h(r)dt^2 + \frac{dr^2}{h(r)} + r^2 d\Omega_4^2\,, \tag{4.1}$$

where the metric coefficient is given by

$$h(r) = q + \frac{r^2}{12\alpha}\left(1 \pm \sqrt{1 + \frac{12\alpha}{5}\Lambda_6 + \frac{24\alpha}{r^5}\mu}\right)\,, \tag{4.2}$$

where q represents the curvature of the four dimensional subspace labeled by $d\Omega_4$, and μ is a mass parameter [5]. From now on, we will consider the solution branch with the minus sign in (4.2): in this case, the limit $\alpha \to 0$ corresponds to an AdS-Schwarzschild geometry of normal Einstein gravity. In the limit $\mu \to 0$, and $q = 0$, one finds the same background discussed in the previous section.

We would like to use this general background to embed a pair of intersecting codimension one branes, and study the cosmology at the intersection along the same lines of the model discussed in the previous section. In order to have a more natural embedding of codimension one branes intersecting at right angle, and to allow a direct comparison with the previously discussed model, it is convenient to change coordinate system in the following way [6]. Defining $\frac{dr}{d\chi} = r\sqrt{h(r)}$, the metric (4.1), for the case $q = 0$, can be re-casted in the form

$$ds_6^2 = -h(\chi)dt^2 + r^2(\chi)\left(\delta_{ij}dx^i dx^j + d\chi^2 + d\eta^2\right)\,. \tag{4.3}$$

Now, let us re-define the extra coordinates as

$$\chi = (1 + k_1 z_1 + k_2 z_2)\,, \qquad \eta = (1 + k_2 z_1 - k_1 z_2)\,,$$

where k_1 and k_2 are two constants satisfying the relation (3.4). We can rewrite the metric (4.3) as

$$ds_6^2 = -B^2(z_1, z_2)dt^2 + A^2(z_1, z_2)\left(\delta_{ij}dx^i dx^j + dz_1^2 + dz_2^2\right)\,. \tag{4.4}$$

The metric coefficients A and B are defined in terms of h and r as:

$$B^2 \equiv h(z_1, z_2) \quad , \quad A^2 \equiv (k_1^2 + k_2^2)r^2(z_1, z_2)\,.$$

[5] This metric ansatz is also suitable to describe solutions for different systems, containing for example antisymmetric forms [22].

[6] We will use again the coordinate system of (4.1) at the end of the next subsection, since it allows to present the results in a particularly transparent form.

For the limiting case $\mu = 0$, it is easy to see that this form of the metric reproduces (3.3), that is,

$$A = B = \frac{1}{1 + k_1 z_1 + k_2 z_2}\,.$$

4.1 Cosmology at the intersection

Starting with the metric (4.4), it is straightforward to embed a pair of codimension one branes. Similarly to what we have done in Section (3.2), we ask that one four brane Σ_1 sits on the line $z_1 = 0$, the fixed point of an orbifold symmetry. The starting bulk metric is consequently

$$ds_6^2 = -B^2(|z_1|, z_2)dt^2 + A^2(|z_1|, z_2)\left(\delta_{ij}dx^i dx^j + dz_1^2 + dz_2^2\right)\,. \tag{4.5}$$

At this point, we introduce a second, dynamical four brane Σ_2, that intersects with a right angle the first one, and moves through the bulk. The induced metric on Σ_2 results to be

$$ds_{\Sigma_2}^2 = -\left[B^2\Big(|z_1|, z_2(t)\Big) - A^2\Big(|z_1|, z_2(t)\Big)\dot{z}_2^2\right]dt^2 + A^2\Big(|z_1|, z_2(t)\Big)\left(\delta_{ij}dx^i dx^j + dz_1^2\right) \tag{4.6}$$

where the dot denotes the derivative with respect to t. Then, the bulk space for $z_2 < z_2(t)$ is identified with the one for $z_2 > z_2(t)$ by a Z_2 symmetry.

The evaluation of the junction conditions for the four branes and for the three brane at the intersection corresponds to a straightforward generalization of calculations developed in Section (3.2.1): in this case, one generally finds that the energy density on the four brane should be anisotropic, since it depends on the coordinate z_1. In the present paper, we limit our analysis to the results for the evolution equations of the three brane at the intersection.

The induced metric on the three brane takes of the FRW form

$$ds_{3+1}^2 = -d\tau^2 + R^2(\tau)\left(\delta_{ij}dx^i dx^j\right)\,, \tag{4.7}$$

where the scale factor again depends on the position of the three brane in the background, and is given by

$$R(\tau) \equiv (k_1^2 + k_2^2)^{\frac{1}{2}}\, r(\tau) = A\Big(0, z_2\Big(t(\tau)\Big)\Big)\,. \tag{4.8}$$

The proper time is connected to the original time t via the definition

$$d\tau = \sqrt{B(0, z_2)^2 - A(0, z_2)^2\,\dot{z}_2^2}\,dt\,.$$

The equations that rule the cosmological evolution for the system are the equation of conservation of energy, and the Friedmann equation. We consider a three brane energy momentum tensor of the perfect fluid form

$$S^{\mu}{}_{\nu} = \mathrm{diag}(-\rho, p, p, p)\,.$$

The standard continuity equation is *not* satisfied in the present case:

$$\frac{d\rho}{d\tau}+3\frac{1}{R(\tau)}\frac{dR(\tau)}{d\tau}(\rho+p)=\rho\frac{d}{d\tau}\ln\frac{\sqrt{h(r(\tau))}}{(k_1^2+k_2^2)^{\frac{1}{2}}r(\tau)}\,, \tag{4.9}$$

where we express this, as well as the following results, in terms of the metric function $h(r)$ as defined in eq. (4.2). It is clear that in this frame a flow of energy leaves the intersection where the three brane is located. Notice that in the case $\mu=0$,

$$\sqrt{h(r)}\propto r\,, \tag{4.10}$$

the right hand side of (4.9) vanishes, and we recover the result of pure AdS. Interestingly, starting from the expression for $h(r)$, one can render the right hand side of (4.9) small enough imposing bounds on the size of μ, and/or requiring that the size of the scale factor is large enough.

Similar features occur discussing the Friedmann equation, that in this model results to be (remember the linear relation between R and r, given in eq. (4.8))

$$H^2\equiv\left(\frac{1}{R}\frac{dR}{d\tau}\right)^2=k_2^2\left[\frac{R^2}{h(r)}\left(\frac{\rho}{\lambda}\right)^2-\frac{h(r)}{R^2}\right]\,, \tag{4.11}$$

where we use eq. (3.9) to define the constant λ. A static configuration is obtained choosing, for example, $\rho=\lambda$, and tuning the scale factor (that is, the position of the brane in the bulk) in such a way that the right hand side of (4.11) vanishes. Next, cosmological evolution is induced perturbing the static configuration with $\rho=\lambda+\rho_3$, and writing

$$H^2=k_2^2\frac{R^2}{h(r)}\frac{1}{\lambda}\left[2\rho_3+\frac{\rho_3^2}{\lambda}+\lambda-\lambda\left(\frac{h(r)}{R^2}\right)^2\right]\,. \tag{4.12}$$

In this way, one recovers a linear term in the energy density plus corrections. Notice that, in this case, the effective Planck mass in four dimensions depends on the value of the scale factor (unless $\mu=0$), being given by

$$M_{Pl}^2=\frac{\lambda}{6k_2^2}\frac{h\left(r(\tau)\right)}{R^2(\tau)}\,. \tag{4.13}$$

In this case the same arguments presented in the case of the equation of continuity are still valid. In particular, one can render the variation rate of the Planck mass, and the size of the corrections in the Friedmann equation, small enough imposing bounds on the size of μ. Let us consider, for example, an expansion of the Friedmann equation for small energy densities and small μ parameter. With $p=-\lambda+p_3$ and $p_3=\omega_3\rho_3$, we get

$$H^2\simeq\frac{2k_2^2}{\lambda}\bar{\rho}_3+\frac{2(1+6\omega_3)k_2^2}{5(-2+3\omega_3)}\frac{\tilde{\mu}}{r^5}+\mathcal{O}\left(\frac{\bar{\rho}_3^2}{\lambda}\right)+\mathcal{O}\left(\frac{\tilde{\mu}}{r^5}\bar{\rho}_3\right)+\mathcal{O}\left(\frac{\tilde{\mu}^2}{r^{10}}\right) \tag{4.14}$$

where we define

$$\bar{\rho}_3 \equiv \rho_0 R^{-3(1+\omega_3)}, \quad \text{with constant } \rho_0, \tag{4.15}$$

$$\tilde{\mu} \equiv \frac{5}{2} \frac{\mu}{(k_1^2 + k_2^2)\sqrt{1 + 12\alpha\Lambda_6/5}}. \tag{4.16}$$

Therefore, we find that the correction due to the nonzero μ reflects a form of dark radiation on the moving 4-brane.

Let us end commenting on the exact forms for the evolution equations, (4.9) and (4.11), that we obtain in this model. These equations are written in such a way that they are valid for *any* bulk metric of the form (4.1), and point out interesting differences between the results of our approach, and typical results found in codimension one brane worlds. For example, the factor of the ρ^2 term in eq. (4.11), related to the Planck mass in four dimensions, depends in general not only on the volume of the extra dimensions, like in the codimension one case, but also on the position of the brane in the bulk. A similar observation regards the non vanishing flow of energy density from the brane to the background: in this frame, we obtain the standard continuity equation only in the case of pure AdS_6 background. However, notice that it is straightforward to Weyl rescale the metric to another frame, where we can recover a standard continuity equation.

5. Conclusions

We studied the cosmological properties of codimension two three brane worlds that sit at the intersection between two codimension one four branes, in the framework of six dimensional Einstein-Gauss-Bonnet gravity. The full backreaction of the defects in the background is completely taken into account. We showed that the junction conditions force the presence of localized energy density on the codimension two defect: the induced metric on this surface assumes a FRW form, with a scale factor associated to the position of the brane in the background. We presented the cosmological evolution equations for the three brane, showing that, for the case of pure AdS_6 backgrounds, they acquire the same form that one finds in the Randall-Sundrum II model. In particular, we can embed on the codimension two defect the preferred form of energy density. The fact that we obtain standard cosmology at late times indicates that gravity on the brane behaves as four dimensional at large distances, with a Planck mass that is proportional to the Gauss-Bonnet parameter α. These properties are obtained at the price of fine tuning relations between the energy density localized on the three brane, and energy that sits on one of the four brane. When the background is different from pure AdS_6, the cosmological behavior is potentially modified in respect to the typical one of codimension one brane worlds, due to the presence of additional contributions to the evolution equations. These contributions depend on the geometrical properties of the background, and

in general describe a flow of energy from the brane to the background. We discussed, in a particular model, the conditions one must satisfy to obtain standard cosmology at late epochs.

Although we considered examples in which the background geometry is controlled just by negative cosmological constant, our approach is completely general, and can be used to embed codimension two brane worlds in vacua for Einstein-Gauss-Bonnet gravity coupled to other fields. This suggests the possibility that a coupling between brane matter and background fields, like for instance bulk scalars, modify the evolution equations, for example compensating the energy flow from brane to bulk, and could also indicate a possible way out for the fine tuning problems we discussed.

Moreover, in principle, a suitable generalization of our approach to systems that contain higher order terms in curvature invariants, can provide a framework to study cosmology for brane worlds in codimension higher than two.

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